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Technical Note

Validation of the local thermal equilibrium assumption in natural convection from a vertical plate embedded in porous medium: non-Darcian model

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Abstract

The validity of the local thermal equilibrium assumption in natural convection over a vertical flat plate embedded in porous medium is investigated analytically. The study is based on the two-phase (Schumann) model, using the Brinkman term (no-slip condition) to cover the flow. It is found that there are four dimensionless parameters controlling the local thermal equilibrium assumption: the volumetric Biot number (Bi_v), the modified Rayleigh number (Ra_m), the modified Darcy number (Da) and the ratio of effective to dynamic viscosity (λ). The effects of these parameters are investigated and a correlation equation is developed in order to determine the region where the local thermal equilibrium assumption is valid.

Keywords: Porous media; Two phase; Non-Darcian; Laminar flow; Natural convection

1. Introduction

In recent years, the convection heat transfer over a vertical flat plate embedded in porous medium has become of great interest and development. This is due to its wide-spread applications, e.g. porous-flat plates collectors, geothermal energy utilization, insulation of nuclear reactors, food storage, filtering, enhanced recovery of petroleum resources, and many other applications [1,2].

During the last few decades, convection heat transfer in porous media has been extensively investigated. Two models are adopted to describe the thermal behavior of porous systems. These are the so-called single-phase and the two-phase (Schumann) models. The main distinction between these two models is that local thermal equilibrium is assumed in the single-phase model while no such assumption is made in the two-phase model. Therefore, the single-phase model yields only one energy equation, whereas in the two-phase model there are two energy equations. In the two-phase model, each energy equation contains a fluid to solid heat transfer term. The concept of local thermal equilibrium has been widely used in modeling transport phenomena in porous media [3–5]. Many investigations in the literature have released the local thermal equilibrium assumption [2,6]. Comparatively, fewer investigations have compared results by the two models [7,8]. Also, there are mainly two models used in the literature to describe the fluid flow in porous medium: one of them is the non-Darcian model [3,4].

The forced convection heat transfer problem in porous medium has been investigated by Al-Nimr and Kiwan [9]. The validity of the local thermal equilibrium assumption in transient conjugated forced convection channel flow has been investigated analytically by Al-Nimr and Abu-Hijleh [10]. The study was focused on the time required for both the solid and fluid to attain approximately the same temperature, and consequently the local thermal equilibrium assumption can be insured.

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Nomenclature

Da	Darcy number $\left(=\frac{h_v K}{\varepsilon k_f}\right)$
f	similarity stream function
g	gravitational acceleration
h	convection heat transfer coefficient
K	permeability
k	thermal conductivity
Bi	Biot number $\left(=\frac{h_v K}{(1-\varepsilon)k_s}\right)$
Ra	Rayleigh number $\left(=\frac{g\beta\Delta T K^{3/2}}{2\alpha_f \varepsilon \upsilon_f}\right)$
T	temperature
u, v	volume-averaged velocity in the (x, y) direction, respectively $\left(u=\frac{\bar{u}}{u_o}, v=\frac{\bar{v}}{u_o}\right)$
x, y	Cartesian coordinate $\left(x=\frac{\bar{x}}{x_o}, y=\frac{\bar{y}}{x_o}\right)$

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ϵ	porosity
η	similarity variable $\left(=\frac{y}{\sqrt{x}}\right)$
Θ	similarity non-dimensional temperature $(=\theta)$
θ	non-dimensional temperature $\left(=\frac{T-T_\infty}{T_w-T_\infty}\right)$
λ	ratio of effective viscosity to dynamic viscosity $\left(=\frac{\tilde{\mu}}{\mu}\right)$
μ	dynamic viscosity
$\tilde{\mu}$	effective viscosity (Brinkman)
υ	kinematic viscosity
Ψ	stream function $(=\sqrt{x}f(\eta))$

Subscripts

f	fluid
m	modified
o	reference
s	solid
v	volumetric
w	wall (plate)
∞	free-stream condition

Superscripts

$'$	differentiation with respect to (η)
-	dimensional quantity

On the other hand, the problem of free convection from vertical flat plate in porous medium has been investigated by many researchers [1,4–6]. In particular, the problem of free convection from vertical plate in porous medium using Darcy model is investigated numerically by Mohamad [6] using the two-phase approach. It was found that the porosity of the medium influences the temperature difference between the solid and fluid within the system. Rees and Pop [2] used a different approach to investigate the same convective boundary layer flow of Mohamad [6]. They used asymptotic analysis to investigate the regions away from leading edge, and to emphasize that at increasing distances from the leading edge the difference between the temperatures of the fluid and solid phases decreases to 0. Under the local thermal equilibrium assumption, the analytical (asymptotic analysis) and the numerical solutions were in agreement for two different kinds of thermal boundary conditions: (1) isothermal wall case and (2) isoflux wall case. This was previously confirmed by Kim and Vafai [5].

It is necessary to identify regions within which the local thermal equilibrium assumption is valid. Al-Nimr and Abu-Hijleh [10] have studied this for the case of transient forced convection from porous channel flow. However, to the best of the authors' knowledge, the case of natural convection from a vertical flat plate embedded in porous medium is not considered in the literature yet. This flow problem is the focus of this paper.

The aim of this study is to investigate the validity of the local thermal equilibrium assumption in the case of free convection flow over an isothermal flat plate embedded in a porous medium. This is accomplished by comparing the results obtained from the single-phase and the two-phase (Schumann) models at different modified Darcy number, volumetric Biot number, modified Rayleigh number, and the ratio of effective to dynamic viscosity. The flow is modeled using Brinkman-extended Darcy model. Regions where the local thermal equilibrium assumption is valid will be identified. The results of this mapping process will be presented in a map form. In addition, these maps will then be represented in a correlation equations form.

2. Mathematical formulation

A two-dimensional, steady, laminar, buoyant flow over a vertical flat plate embedded in a saturated homogenous porous medium is considered. The fluid is considered incompressible and Newtonian with constant physical properties except the density in the buoyancy term (Boussinesq approximation). In addition, the fluid is modeled by the Brinkman-extended Darcy model.

The origin of the x–y Cartesian coordinate is attached at the leading edge of the vertical plate with the x-axis along the upward flat plate direction and the y-axis is in the transverse direction. Now, defining the stream function Ψ such that $\bar{u} = \partial\Psi/\partial\bar{y}$ and $\bar{v} = -\partial\Psi/\partial\bar{x}$, the governing equations in dimensionless form are given as:

Momentum equation

$$\lambda\frac{\partial^4\Psi}{\partial y^4} = \frac{\partial^2\Psi}{\partial y^2} - \frac{\partial\theta_f}{\partial y} \tag{1}$$

Energy equation-solid phase

$$\frac{\partial^2\theta_s}{\partial y^2} = Bi_v(\theta_s - \theta_f) \tag{2a}$$

Energy equation-fluid phase

$$\frac{\partial^2\theta_f}{\partial y^2} = Da(\theta_f - \theta_s) + 2Ra_m\left(\frac{\partial\Psi}{\partial y}\frac{\partial\theta_f}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial\theta_f}{\partial y}\right) \tag{2b}$$

where $x = \bar{x}/x_o$, $y = \bar{y}/x_o$, $u = \bar{u}/u_o$, $v = \bar{v}/u_o$, $\theta = (T - T_\infty)/(T_w - T_\infty)$, $\Delta T = (T_w - T_\infty)$, $x_o = \sqrt{K}$, $\lambda = \tilde{\mu}/\mu$, $u_o = g\beta K\Delta T/\upsilon_f$, $Ra_m = g\beta\Delta TK^{3/2}/2\alpha_f\varepsilon\upsilon_f$, $Bi_v = h_vK/(1-\varepsilon)k_s$, $Da = h_vK/\varepsilon k_f$.

Also, the dimensionless forms of the boundary conditions are given as:

$$\begin{aligned}&\text{at } y = 0: \ u, v = 0, \quad \theta_f = \theta_s = 1,\\ &\text{at } y \to \infty: \ u = \frac{\partial u}{\partial y} = 0, \quad \theta_f = \theta_s = 0\end{aligned} \tag{3}$$

The resulting governing non-dimensional equations given above are transformed into local similarity equations by introducing the following variables:

$$\eta = \frac{y}{\sqrt{x}}, \quad \Psi(\eta) = \sqrt{x}f(\eta), \quad \theta_s = \Theta_s(\eta),$$
$$\theta_f = \Theta_f(\eta) \tag{4}$$

Substituting these back into Eqs. (1) and (2) produces the following locally similar equations:

$$\lambda f^{IV} = x(f'' - \Theta_f') \tag{5a}$$

$$\Theta_s' = xBi_v(\Theta_s - \Theta_f) \tag{5b}$$

$$\Theta_f'' = xDa(\Theta_f - \Theta_s) - Ra_m f\Theta_f' \tag{5c}$$

Similarly, upon using the definitions introduced in Eq. (4), the boundary conditions given by Eq. (3) become:

$$\begin{aligned}&@\eta = 0: \ f = f' = 0, \quad \Theta_s = \Theta_f = 1,\\ &@\eta \to \infty: \ f' = f'' = 0, \quad \Theta_s = \Theta_f = 0\end{aligned} \tag{6}$$

3. Solution methodology

The governing system of ordinary differential equations (5) subjected to the boundary conditions in Eq. (6) forms a boundary value problem, which can be solved by many well-established numerical techniques. In this study, the finite difference method with non-uniform grid is used to solve this system of equations. A FORTRAN computer program is developed for this purpose which makes use of BVPFD-subroutine. BVPFD is a general purpose subroutine in the IMSL library which is designed to solve boundary value problem governed by a system of ordinary differential equations using the finite difference method.

At first, the solution domain has to be identified, and the domain of integration in the wall normal η-direction has to be finite. It was founded that the infinity is located beyond 4.5. Thus, the boundary location is considered at $\eta = 5$ for the rest of this study.

In the present work, the local thermal equilibrium assumption is assumed to be valid when the absolute difference in temperature between the solid and fluid phases ($T_f - T_s$) is less than or equal to 1% of $T_w - T_\infty$, i.e. when

$$|\theta_s - \theta_f| \leqslant 0.01$$

At each x-location along the plate, there will be a corresponding η that divides the domain into local and non-local thermal equilibrium domains. When these border points are connected along the plate, regions of local and non-local thermal equilibrium can be easily identified. This procedure was repeated for a wide range of flow parameters Bi_v, Ra_m, Da, and λ. As a result, the aspects of the effect of each parameter are revealed.

In an attempt to make reading of the results easier, the obtained results are fitted numerically by using the least squares method. The resulting correlations could be used to check the validity of using or not using the local thermal equilibrium assumption.

4. Results and discussion

The effect of modified Darcy number, volumetric Biot number, modified Rayleigh number and the ratio of effective to dynamic viscosity is investigated, and the results are shown in Figs. 1–4. Figs. 1–4 represent a mapping for the region within which the local thermal equilibrium assumption is unjustified. In these regions, the difference between the solid and fluid temperatures is relatively significant (i.e. $|\theta_s - \theta_f| > 0.01$), so that the two-phase (Schumann) model is reasonable to be used. It can be noted that the maps (curves) shown in Figs. 1–4 can be divided into three-distinct regions—(1) region 1: in this region, x is always greater than $x_{plateau}$, where $x_{plateau}$ is the x-value of the plateau part of the curve. In this region, the local thermal equilibrium assumption is acceptable for the entire range in the transverse direction y. (2) Region 2: in this region x is less than $x_{plateau}$, and the product of $\eta\sqrt{x}$ is greater than the corresponding value on the curve. In this region, the temperature

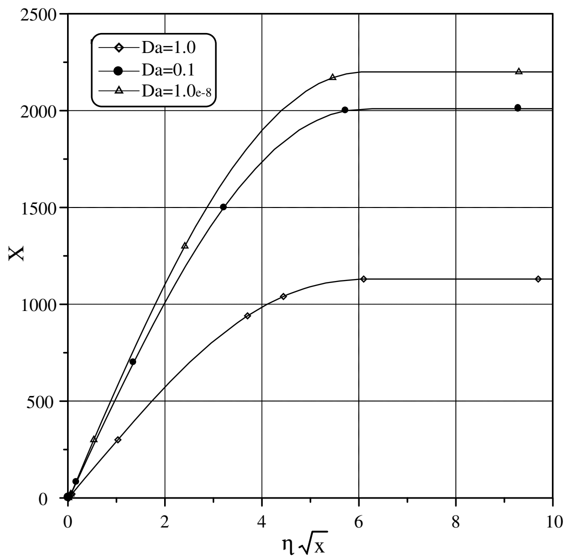

Fig. 1. Effect of modified Darcy number on the validity of the local thermal equilibrium model ($Ra_m = 100$, $\lambda = 1.0$, and $Bi_v = 1.0$).

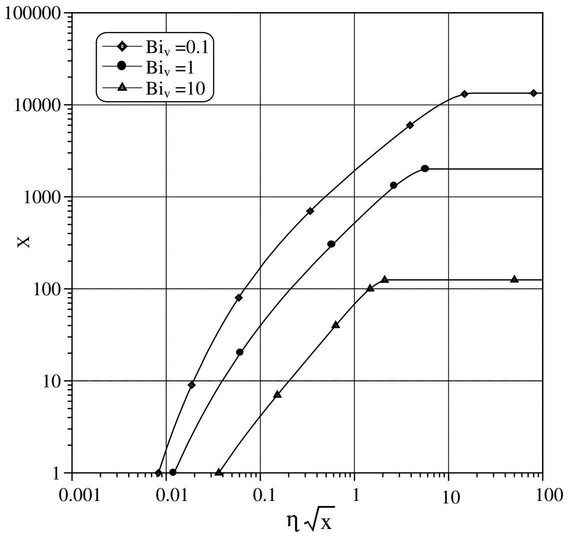

Fig. 2. Effect of volumetric Biot number on the validity of the local thermal equilibrium model ($Ra_m = 100$, $\lambda = 1.0$, and $Da = 0.1$).

difference between the solid and fluid phases is significant, and hence the two-phase (Schumann) model must be used. (3) Region 3: in this region x is less than x_{plateau}, and the product of $\eta\sqrt{x}$ is less than the corresponding value on the curve. In this region, the local thermal equilibrium assumption is valid as in region 1.

The effect of Da on the region within which the local thermal equilibrium assumption is valid is shown in Fig.

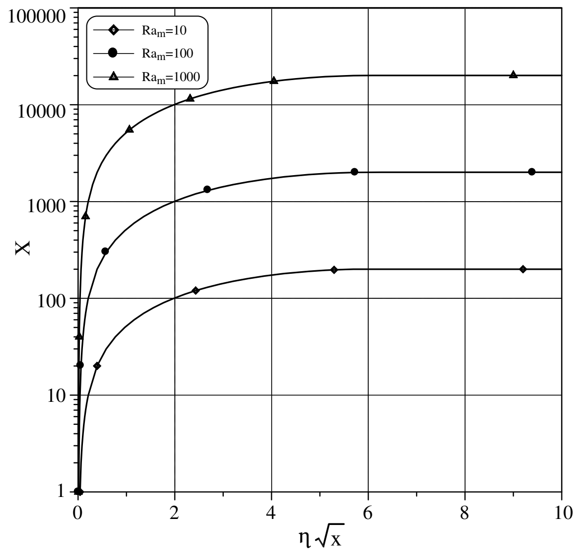

Fig. 3. Effect of modified Rayleigh number on the validity of the local thermal equilibrium model ($Da = 0.1$, $\lambda = 1.0$, and $Bi_v = 1.0$).

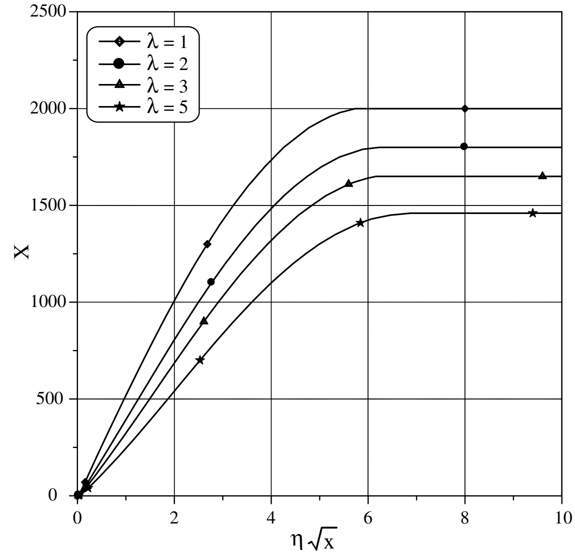

Fig. 4. Effect of effective viscosity to dynamic viscosity ratio on the validity of the local thermal equilibrium model ($Ra_m = 100$, $Da = 0.1$, and $Bi_v = 10$).

1, for the range ($1.0\text{e}{-8} \leqslant Da \leqslant 1$). Obviously, as Da increases, region 1 within which the local thermal equilibrium assumption is valid increases. Also, the range in the wall normal direction within which the local thermal equilibrium assumption is valid (i.e. region 3) increases. This can be attributed to the fact that as Da increases, the porous medium becomes lighter and thus the volumetric surface area increases. As a result, the convection heat transfer between the solid and fluid phases increases. This in turn enforces the local thermal equilibrium assumption. In addition, increasing Da will ease the motion of the fluid within the pores due to the increase in the permeability of the porous medium K. As a result, the heat transfer between the fluid and solid phases will be enhanced, and this enforces the local equilibrium assumption too.

Similarly, the Bi_v effect on the local thermal equilibrium assumption is investigated. The result is shown in Fig. 2, for the range $0.1 \leqslant Bi_v \leqslant 10$. It is clear that as Bi_v increases, region 1 within which the local thermal equilibrium assumption is valid increases. Also, the range in the wall normal direction within which the local thermal equilibrium assumption is valid (i.e. region 3) increases. To explain this we recall that Bi_v is proportional to the permeability K of the porous medium and to the volumetric convection heat transfer coefficient between the fluid and solid matrix h_v. By increasing the permeability of the medium, the flow of the fluid within the medium becomes easier. This leads to enhanced heat transfer between the solid and fluid and thus enforces the local thermal equilibrium assumption. In the meantime, increasing h_v will increase the heat transfer between the solid and fluid phases, and this enforces the local equilibrium assumption too.

Fig. 3 shows the effect of Ra_m on the regions within which the two-phase (Schumann) model is significant for the wide range of $10 \leqslant Ra_m \leqslant 1000$. It can be noted that as Ra_m increases, region 1 within which the local thermal equilibrium assumption is valid decreases. Also, the range in the wall normal direction within which the local thermal equilibrium assumption is valid (i.e. region 3) decreases. The buoyancy force may be decreased by decreasing the temperature difference between the wall and free-stream. This leads to a reduction in the traveling distance required by the flow along the plate to attain the local equilibrium all over the domain in the transverse direction. This enforces the local thermal equilibrium assumption. On the other hand, increasing the fluid's thermal diffusivity will increase heat conduction in the wall normal direction. This enhances the heat transfer between the fluid and solid matrix, and thus enforces the local equilibrium assumption. It is worth mentioning here that at large value of Ra_m ($\geqslant 1000$), the effect of the inertial force may be significant. As a result, one should be careful in using the Ra_m map within high ranges. At large values of Ra_m, the microscopic Forcheimer inertial term must be included in the momentum equation.

Lastly, the effect of λ $(\tilde{\mu}/\mu)$ on the region within which the local equilibrium assumption is valid is shown in Fig. 4, for the range $0.1 \leqslant \lambda \leqslant 10$. Fig. 4 displays that as the effective viscosity (relatively to the dynamic viscosity) increases, the regions within which the local equilibrium assumption is valid (regions 1 and 3) increase. This helps the fluid in attaining thermal equilibrium faster. For an isotropic porous medium, λ is proportional to $1/\varepsilon T^*$, where T^* represents the tortuosity of the porous medium. Many investigations assume that $T^* = 1$, and thus $\lambda = 1/\varepsilon$. Hence, λ depends on the geometry and on the porosity of the medium [1, p. 13]. Thus decreasing the medium porosity will increase the convection contact area, leading to increased heat transfer between the fluid and solid. This also helps the fluid in attaining thermal equilibrium faster and thus enforces the local equilibrium assumption.

As a result, two correlation equations are obtained based on the mapping results discussed above. These equations are valid for the specified ranges with a reasonable error:

(1) To estimate the length along the plate x_{plateau} beyond which the local thermal equilibrium assumption is valid everywhere in the flow domain:

$$x_{\text{plateau}} = \frac{16 Ra_m^{1.000}}{\lambda^{0.157} Bi_v^{1.07} Da^{0.022}} \tag{7}$$

with maximum error of 16% for $Da \leqslant 0.1$, $Bi_v \geqslant 1$, $Ra_m \geqslant 10$, and $\lambda \geqslant 1.0$. The above correlation implies that the local thermal equilibrium assumption is valid within regions having $x > x_{\text{plateau}}$ and $y \geqslant 0$.

(2) On the other hand, the region within which using the two-phase (Schumann) model is essential, is given as:

$$x < x_{\text{Plateau}}$$
$$y = \frac{Da^{0.017} Bi_v^{0.535} \lambda^{0.321} X^{0.844}}{2.72405 Ra_m^{0.832}} \tag{8}$$

where x_{plateau} is given in Eq. (7). Correlation (8) has a maximum error of 30% for $Da \leqslant 0.1$, $Bi_v \geqslant 1$, $Ra_m \geqslant 10$, $\lambda \geqslant 1$, and $x > x_{\text{plateau}}/2$.

5. Conclusions

In this study, the validity of the local thermal equilibrium assumption is checked for the case of natural convection heat transfer from a vertical flat plate embedded in porous medium. The study is based on the Brinkman-extended Darcy model and it was assumed that the solid and fluid phases are not in local thermal equilibrium due to heat transfer between them.

It is found that there are four dimensionless parameters that govern the validity of the local equilibrium assumption. These parameters are Bi_v, Da, Ra_m, and λ.

The conclusions from this study can be summarized by the fact that local thermal equilibrium could be assumed with sufficient accuracy, and that this assumption is valid for high value of Da, Bi_v, and λ, and lower values of Ra_m. This implies that the assumption is not justified for relatively low values of Da, Bi_v, and λ, and high values of Ra_{m}. In such conditions, the two-phase (Schumann) model is recommended.

The quantitative description of these conclusions is presented in correlation equations, for more convenience.

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